Lecture 4

March 11, 2016

1 Image distortions at first order

Circular sources are mapped into elliptical sources. We can build a toy model to show this. Let's use a Sersic model to represent the surface brightness of our source:

$$I(r) \sim \exp\left[-b(n)\left(\frac{r}{r_e}\right)^{1/n} - 1\right]$$

Lens mapping at first order implies that we are mapping from one plane to the other using the equations:

$$\beta_1 = A_{11}\theta_1 + A_{12}\theta_2$$
$$\beta_2 = A_{22}\theta_2 + A_{12}\theta_1$$

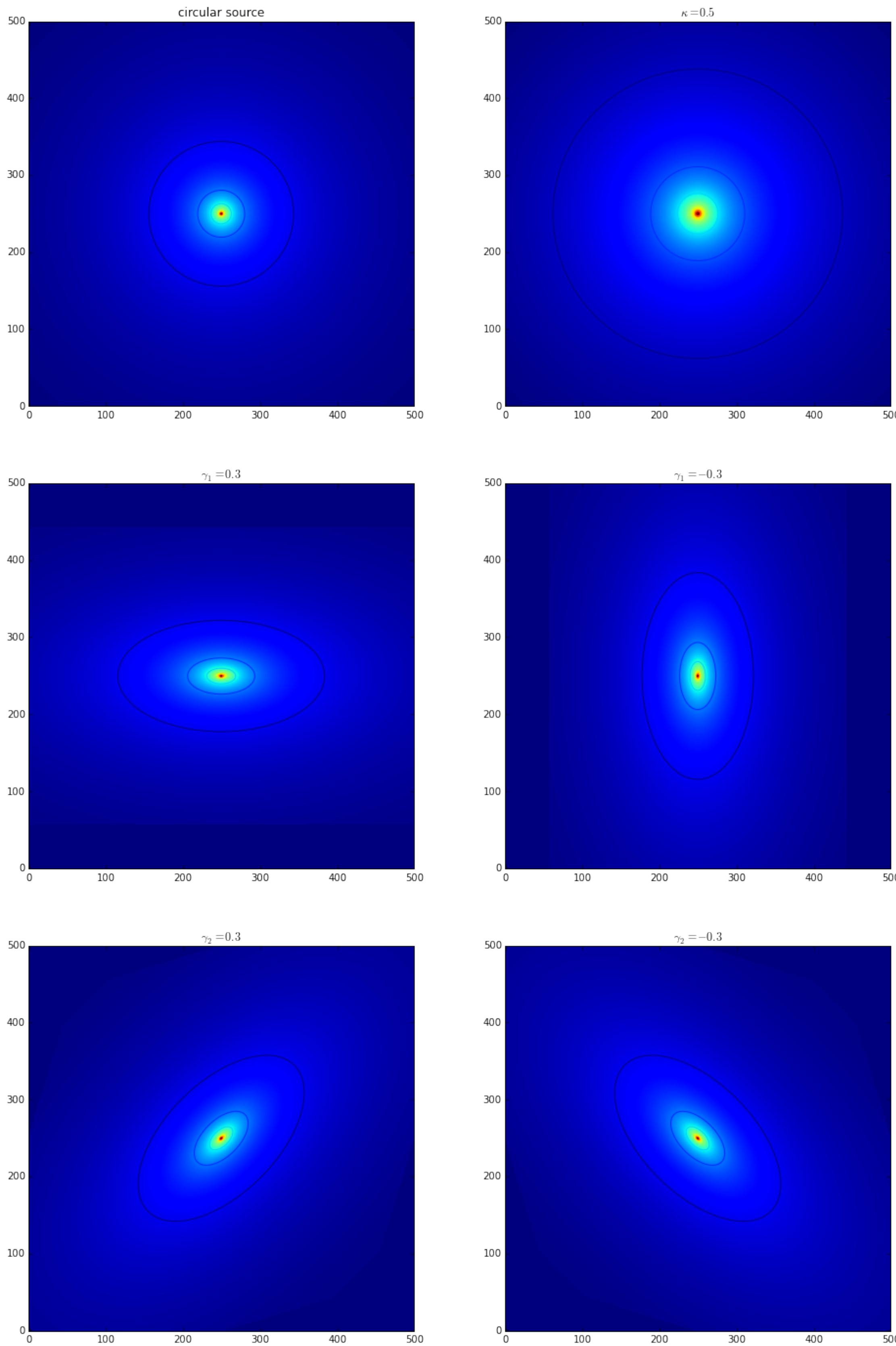
In [1]:
```
import matplotlib.pyplot as plt
%matplotlib inline
import galflex


# build a circular reference galaxy with a Sersic profile
gal1 = galflex.Sersic(3.0, 4, N=500, flux=10.0, q=1.0, phi=0.0, cx=0.0,cy=0.0, re=60.0)

# build another identical galaxy with which we will play

gal = galflex.Sersic(3.0, 4, N=500, flux=10.0, q=1.0, phi=0.0, cx=0.0,cy=0.0, re=60.0)

gal.lens(kap=0.5, gamma1=0.0, gamma2=0.0)

fig,ax=plt.subplots(3,2,figsize=(16,24))
ax[0,0].imshow(gal1.image,origin='lower')
ax[0,0].contour(gal1.image)
ax[0,0].set_title('circular source')
ax[0,1].imshow(gal.image,origin='lower')
ax[0,1].contour(gal.image)
ax[0,1].set_title('$\kappa=0.5$')

gal = galflex.Sersic(3.0, 4, N=500, flux=10.0, q=1.0, phi=0.0, cx=0.0,cy=0.0, re=60.0)
gal.lens(kap=0.0, gamma1=0.3, gamma2=0.0)
ax[1,0].imshow(gal.image,origin='lower')
ax[1,0].contour(gal.image)
ax[1,0].set_title('$\gamma_1=0.3$')

gal = galflex.Sersic(3.0, 4, N=500, flux=10.0, q=1.0, phi=0.0, cx=0.0,cy=0.0, re=60.0)
gal.lens(kap=0.0, gamma1=-0.3, gamma2=0.0)
```

```
ax[1,1].imshow(gal.image,origin='lower')
ax[1,1].contour(gal.image)
ax[1,1].set_title('$\gamma_1=-0.3$')

gal = galflex.Sersic(3.0, 4, N=500, flux=10.0, q=1.0, phi=0.0, cx=0.0,cy=0.0, re=60.0)
gal.lens(kap=0.0, gamma1=0.0, gamma2=0.3)
ax[2,0].imshow(gal.image,origin='lower')
ax[2,0].contour(gal.image)
ax[2,0].set_title('$\gamma_2=0.3$')

gal = galflex.Sersic(3.0, 4, N=500, flux=10.0, q=1.0, phi=0.0, cx=0.0,cy=0.0, re=60.0)
gal.lens(kap=0.0, gamma1=0.0, gamma2=-0.3)
ax[2,1].imshow(gal.image,origin='lower')
ax[2,1].contour(gal.image)
ax[2,1].set_title('$\gamma_2=-0.3$')
```

```
/Users/massimo/anaconda/envs/python2/lib/python2.7/site-packages/matplotlib/collections.py:650: FutureWa
  if self._edgecolors_original != str('face'):
```

Out[1]: <matplotlib.text.Text at 0x1097f2050>

```
/Users/massimo/anaconda/envs/python2/lib/python2.7/site-packages/matplotlib/collections.py:590: FutureWa
  if self._edgecolors == str('face'):
```


circular source
$\kappa = 0.5$
$\gamma_1 = 0.3$
$\gamma_1 = -0.3$
$\gamma_2 = 0.3$
$\gamma_2 = -0.3$

2 Critical lines

Critical lines are defined by the equations

$$\lambda_t = 1 - \kappa - \gamma = 0$$

and

$$\lambda_r = 1 - \kappa + \gamma = 0$$

where $\gamma = (\gamma_1^2 + \gamma_2^2)^{1/2}$.

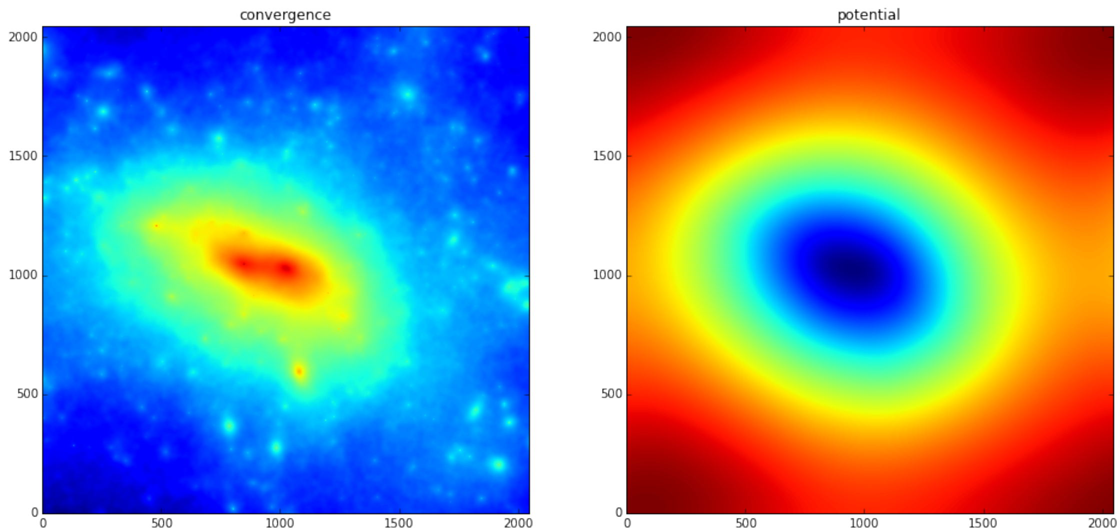
Since the λ_t and λ_r are the eigenvalues of the lensing Jacobian, they can be visualized as the zero-level countours of the $\det A$ map.

In [2]:
```
import numpy as np
from astropy.io import fits as pyfits
#from ..utils.fft import NUMPYFFTPack
#fftengine = NUMPYFFTPack()
import numpy.fft as fftengine
import matplotlib.pyplot as plt
from matplotlib.colors import LogNorm, PowerNorm, SymLogNorm
%matplotlib inline

def potential(kappa,resolution):

    l = np.array(np.meshgrid(fftengine.rfftfreq(kappa.shape[0]),fftengine.fftfreq(kappa.shape[1


    #Compute the magnitude squared of the wavenumber (laplacian in Fourier space = -4*pi*l_squa
    l_squared = l[0]**2 + l[1]**2
    l_squared[0,0] = 1.0

    #Go with the FFTs

    density_ft = fftengine.rfft2(kappa)

    #Invert the laplacian
    density_ft *= -2.0*(resolution)**2 / (l_squared * ((2.0*np.pi)**2))
    density_ft[0,0] = 0.0

    potential=fftengine.irfft2(density_ft) #if resolution is in rad, this is rad**2
    return potential

filekappa='/Users/massimo/stiva/FF/Hera/g0016649_G/yz_nocircles/kappa_2.fits'
kappa,header=pyfits.getdata(filekappa,header=True)

resolution=header["CDELT2"]*3600.0*np.pi/180.0
print resolution,header["CDELT2"]

pot=potential(kappa,resolution)

fig,ax = plt.subplots(1,2,figsize=(16,8))
ax[0].imshow(kappa,origin="lower",norm=LogNorm())
ax[0].set_title('convergence')
```

```
ax[1].imshow(pot,origin="lower")
ax[1].set_title('potential')
```

```
0.00193584939314 3.081e-05
```

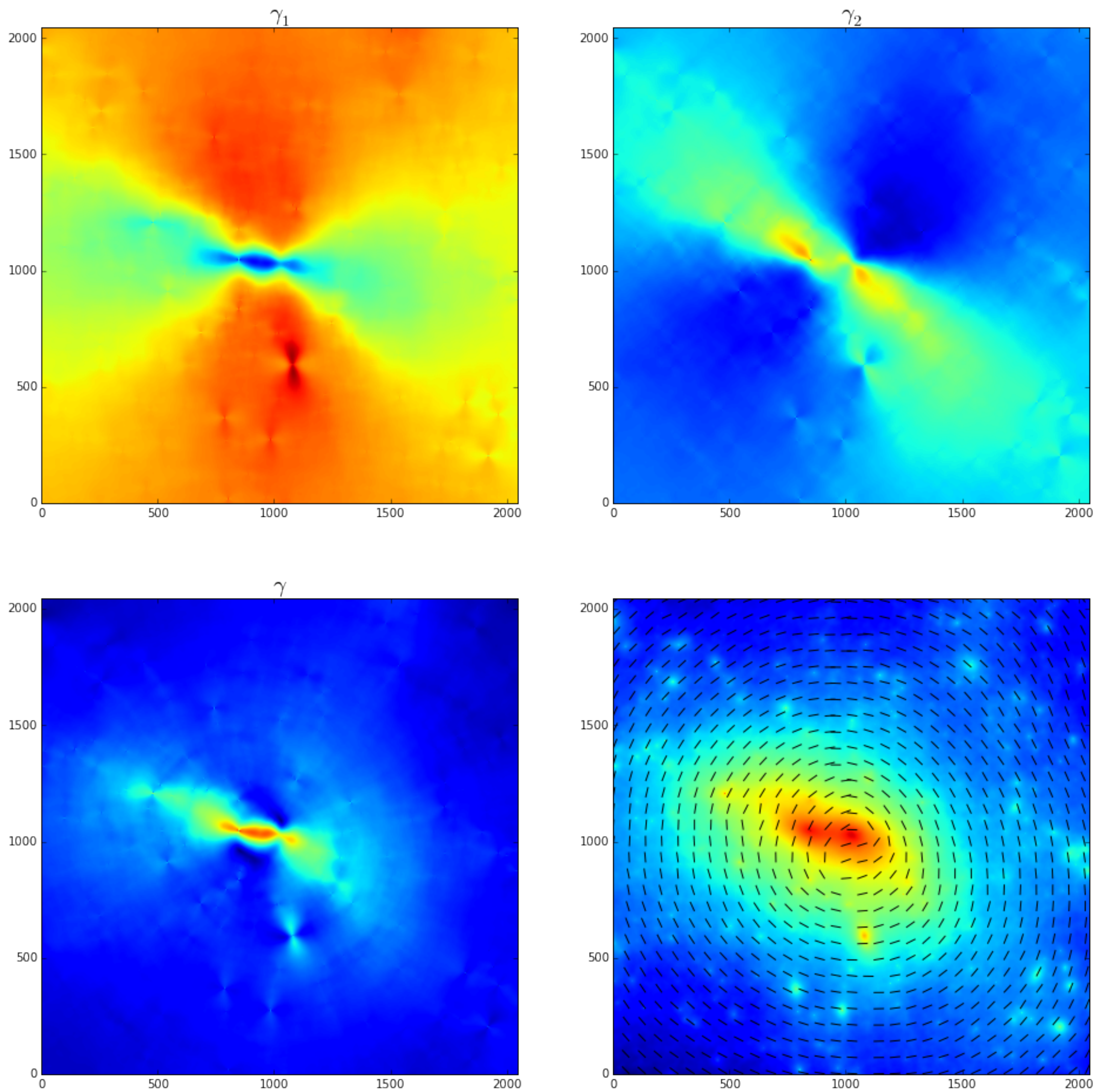
Out[2]: <matplotlib.text.Text at 0x10bcc0cd0>



In [3]:
```
filegamma1='/Users/massimo/stiva/FF/Hera/g0016649_G/yz_nocircles/gammax_2.fits'
filegamma2='/Users/massimo/stiva/FF/Hera/g0016649_G/yz_nocircles/gammay_2.fits'
gamma1,header1=pyfits.getdata(filegamma1,header=True)
gamma2,header2=pyfits.getdata(filegamma2,header=True)
gamma=np.sqrt(gamma1*gamma1+gamma2*gamma2)

fig,ax = plt.subplots(2,2,figsize=(16,16))
ax[0,0].imshow(gamma1,origin='lower')
ax[0,0].set_title('$\gamma_1$',fontsize=20)
ax[0,1].imshow(gamma2,origin='lower')
ax[0,1].set_title('$\gamma_2$',fontsize=20)
ax[1,0].imshow(gamma,origin='lower')
ax[1,0].set_title('$\gamma$',fontsize=20)
ax[1,1].imshow(kappa,origin='lower',norm=LogNorm())

#shear pattern
pixel_step=70
x,y = np.meshgrid(np.arange(0,gamma.shape[1],pixel_step),np.arange(0,gamma.shape[0],pixel_step))

#Translate shear components into sines and cosines
cos_2_phi = -gamma1 / np.sqrt(gamma1**2 + gamma2**2)
sin_2_phi = gamma2 / np.sqrt(gamma1**2 + gamma2**2)

#Compute stick directions
cos_phi = np.sqrt(0.5*(1.0 + cos_2_phi)) * np.sign(sin_2_phi)
sin_phi = np.sqrt(0.5*(1.0 - cos_2_phi))
```

```
#Fix ambiguity when sin_2_phi = 0
cos_phi[sin_2_phi==0] = np.sqrt(0.5*(1.0 + cos_2_phi[sin_2_phi==0]))

#ax[1,0].quiver(x,y,cos_phi[x,y],sin_phi[x,y],headwidth=0,units="height",scale=x.shape[0],color

ax[1,1].quiver(x,y,cos_phi[x,y],sin_phi[x,y],headwidth=0,units="height",scale=x.shape[0],color=
```

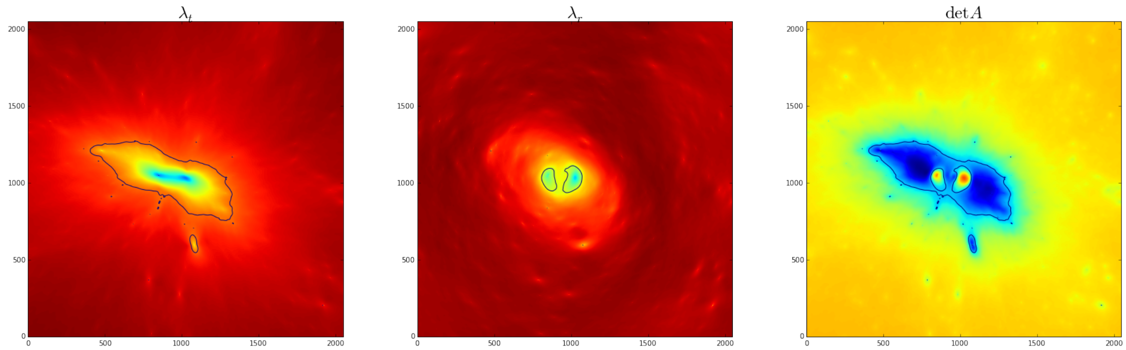
Out[3]: <matplotlib.quiver.Quiver at 0x10e2a1c10>



In [4]:
```
detA=(1.0-kappa-gamma)*(1.0-kappa+gamma)
lambdat=1.0-kappa-gamma
lambdar=1.0-kappa+gamma

fig,ax=plt.subplots(1,3,figsize=(28,8))
ax[0].imshow(lambdat,origin='lower')
ax[0].contour(lambdat,levels=[0.0])
```

```
ax[0].set_title('$\lambda_t$',fontsize=25)
ax[1].imshow(lambdar,origin='lower')
ax[1].contour(lambdar,levels=[0.0])
ax[1].set_title('$\lambda_r$',fontsize=25)
ax[2].imshow(detA,origin='lower',norm=SymLogNorm(0.3))
ax[2].contour(detA,levels=[0.0])
ax[2].set_title('$\det A$',fontsize=25)
```

Out[4]: <matplotlib.text.Text at 0x126413390>



3 Critical lines vs source redshift

We have seen that

$$\hat{\Psi} = \frac{D_{LS}}{D_L D_S} \frac{2}{c^2} \int \Phi dz$$

We have seen also that

$$\kappa, \gamma = f(\hat{\Psi}_{ij})$$

Since each derivative brings in a factor D_L, we have that

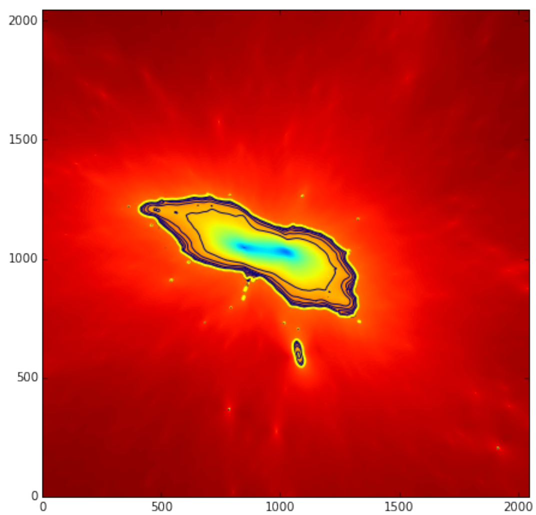
$$\kappa, \gamma \propto \frac{D_{LS} D_L}{D_S} \propto D_{lens}$$

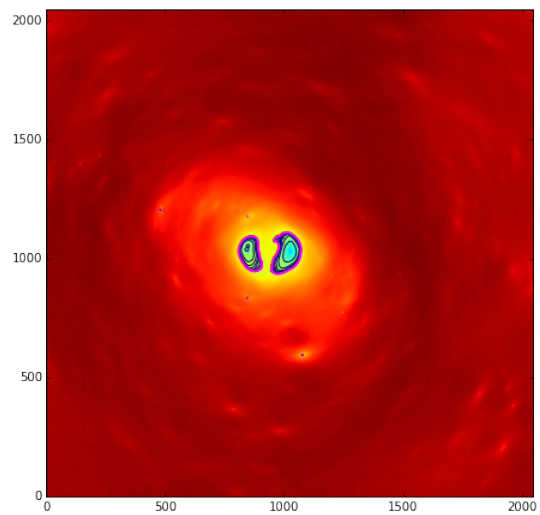
This scaling of κ and γ with distances implies that a given lens has critical lines for each source redshift. The maps above are normalized to a source redshift $z_S = 9.0$. The lens is at redshift $z_L = 0.5$.

In [5]:
```
from astropy.cosmology import FlatLambdaCDM
cosmo = FlatLambdaCDM(H0=70, Om0=0.3)

zl=0.5
zs=np.linspace(zl,10.0,10)
dl=cosmo.angular_diameter_distance(zl)
ds=cosmo.angular_diameter_distance(zs)
dls=[]
for i in range(ds.size):
    dls.append(cosmo.angular_diameter_distance_z1z2(zl,zs[i]).value)

zs_norm=9.0
ds_norm=cosmo.angular_diameter_distance(zs_norm)
```

```
dls_norm=cosmo.angular_diameter_distance_z1z2(zl,zs_norm)

fig,ax=plt.subplots(1,2,figsize=(16,8))
ax[0].imshow(lambdat,origin='lower')
ax[1].imshow(lambdar,origin='lower')
for i in range(ds.size):
    kappa_new=kappa*ds_norm.value/dls_norm.value*dls[i]/ds[i].value
    gamma_new=gamma*ds_norm.value/dls_norm.value*dls[i]/ds[i].value
    lambdat_new=(1.0-kappa_new-gamma_new)
    lambdar_new=(1.0-kappa_new+gamma_new)
    ax[0].contour(lambdat_new,levels=[0.0])
    ax[1].contour(lambdar_new,levels=[0.0])

ax[0].contour(lambdat,levels=[0.0],colors="yellow",linewidths=2)
ax[1].contour(lambdar,levels=[0.0],colors="magenta",linewidths=2)
```

```
Out[5]: <matplotlib.contour.QuadContourSet instance at 0x12667c440>
```





4 Caustics

The caustics are the "sources" of the critical lines. In other words, if $\vec{\theta}_c$ defines a set of points belonging to the critical lines, then

$$\vec{\beta}_c = \vec{\theta}_c - \vec{\alpha}(\vec{\theta}_c)$$

defines a set of points belonging to the caustics.

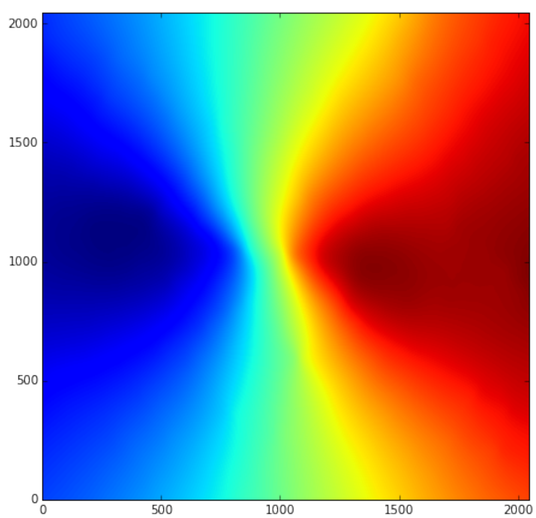
To procede to the calculation of the caustics, we need first to gather the deflection angles. In the following, I will work in pixel units:

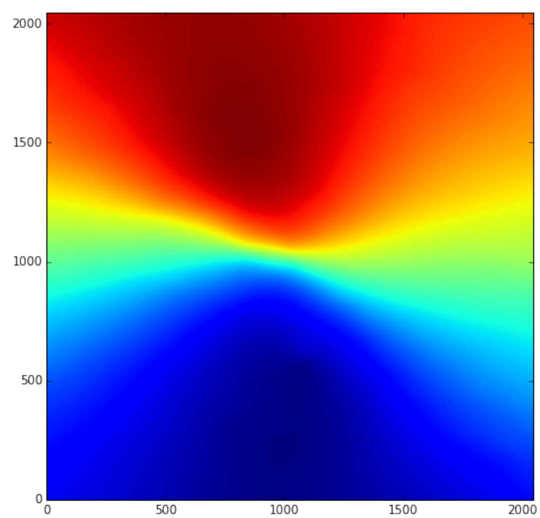
```
In [6]: filealpha='/Users/massimo/stiva/FF/Hera/g0016649_G/yz_nocircles/deflAnglesHera.fits'
        hdu=pyfits.open(filealpha) # read the deflection angle file

        # need to do some manipulation because I wrote it only for zs=2, while this exercise will be fo
        zs_norm=2.0
```

```
zs=9.0
ds_norm=cosmo.angular_diameter_distance(zs_norm)
dls_norm=cosmo.angular_diameter_distance_z1z2(zl,zs_norm)
ds=cosmo.angular_diameter_distance(zs)
dls=cosmo.angular_diameter_distance_z1z2(zl,zs)


# convert the deflection angles in pixel units
resolution=header["CDELT2"]*3600.0
alpha1=hdu[0].data/(resolution)*dls/ds*ds_norm/dls_norm
alpha2=hdu[1].data/(resolution)*dls/ds*ds_norm/dls_norm
# how big are the deflections?
print np.amin(alpha1),np.amax(alpha1)

# plot the two components
fig,ax=plt.subplots(1,2,figsize=(16,8))
ax[0].imshow(alpha1,origin='lower')
ax[1].imshow(alpha2,origin='lower')
```

```
-308.049072266 276.430206299
```

Out[6]: <matplotlib.image.AxesImage at 0x13be46810>



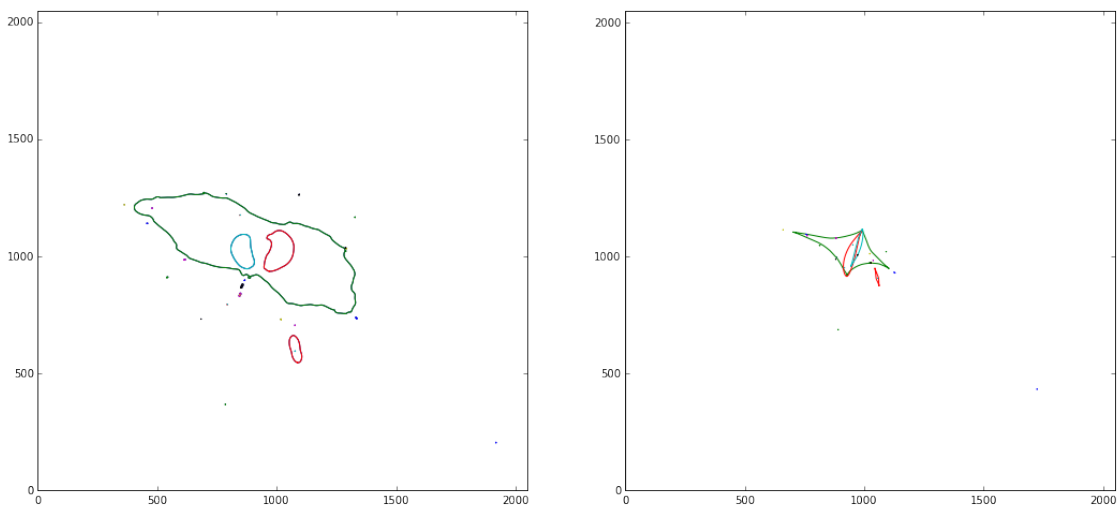


Now that we have the deflection angle maps, we can pick up again the critical lines. We will work with the critical lines for $z_S = 9$, defined above.

In [7]:
```
fig,ax=plt.subplots(1,2,figsize=(18,8))
cs=ax[0].contour(detA,levels=[0.0])

contour=cs.collections[0]
p=contour.get_paths() # p contains the paths of each individual critical line
sizevs=np.empty(len(p),dtype=int)

from scipy.ndimage import map_coordinates

if (sizevs.size > 0):
```

```
    for j in range(len(p)):
        # for each path, we create two vectors containing the x1 and x2 coordinates of the vert
        vs = contour.get_paths()[j].vertices
        sizevs[j]=len(vs)
        x1=[]
        x2=[]
        for i in range(len(vs)):
            xx1,xx2=vs[i]
            x1.append(float(xx1))
            x2.append(float(xx2))
        # these are the points we want to map back on the source plane.
        # To do that we need to evaluate the deflection angle at their positions
        # using scipy.ndimage.interpolate.map_coordinates we perform a bi-linear interpolation
        a1=map_coordinates(alpha1, [[x2],[x1]],order=1)
        a2=map_coordinates(alpha2, [[x2],[x1]],order=1)

        # now we can make the mapping using the lens equation:
        y1=x1-a1[0]
        y2=x2-a2[0]

        # plot the results!
        ax[0].plot(x1,x2,'-')
        ax[1].plot(y1,y2,'-')

ax[1].set_xlim([0,2048])
ax[1].set_ylim([0,2048])
```

Out[7]: (0, 2048)



Left and right panels show the lens and the source planes, respectively. They display exactly the same region of the sky.